AP® Calculus AB
Course Syllabus

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Introduction:

The study of calculus concepts begins with an intuitive introduction through the examination of graphical problems (such as the "tangent line problem" and the "area under the curve problem"), or through numerical data analysis (such as the introduction to limits). Calculus cannot be studied by memorizing formulas or processes.

This course covers the first semester of calculus and prepares students to take the AP ® Calculus AB exam. Students must be fluent at calculus computations and must also be able to apply their knowledge of calculus to solve "real-world" situations. An emphasis is placed on the connection between multiple representations of concepts (graphical, numerical, analytical, and verbal), strengthening students' understanding of the concepts and ability to apply them.

Instructional Material:

- Primary Textbook:
Anton, Howard. *Calculus: A New Horizon*. 6th Ed./Brief Ed. New York: John Wiley & Sons, 1999.

- Technology / Calculator:
Graphing Calculator- TI-83+ or TI-84 is recommended, but any graphing calculator on the AP Exam approved calculator list is accepted.

- Online Resources:
Visit www.morganmath.com for course information including assignments and documents.

Grading:

Categories and percentages are approximate and may change based on actual assignments.

10% Classwork / Projects	90% – 100%	A
20% Homework	80% – 89%	B
50% Tests & Quizzes	70% – 79%	C
20% Final Exam	69% or below	No Credit

Course Outline:

Appendix Review: Preparing for Calculus (2 days)

- Open and closed intervals
- Graphing and solving inequalities and absolute value.
- The Coordinate Plane: lines, intercepts, slope of a line, vertical and horizontal lines, distance between two points, midpoint of two points, quadratic equations, circles, unit circle.
- Scatter plots, tabular data, and extracting information from graphs.
- Trigonometric functions: development from unit circle, radians as no/dimensionless units, trig identities and properties.

Chapter 1: A Review of Functions (3 days)

- Review of families of functions: lines, parabolas, cubic, polynomials, exponential, logarithmic.
- Functions: domain and range, finding domain and range both algebraically and graphically.
- Compare four ways to represent a function: algebraically, graphically, numerically as data/table, and verbally. Discuss strengths and weaknesses of each representation.
- Piecewise functions, absolute value as piecewise.
- Using technology: issues of scale when graphing, choosing viewing window on graphing calculators, and identifying false gaps or connected asymptotes.
- Composition of functions, even/odd functions, translations, symmetry.
- Parametric equations and application

Chapter 2: Limits and Continuity: The Building Blocks of Calculus (6 days)

- Intuitive development of limits: the Tangent Line problem and the Area Under the Curve problem, graphical representations, data/table representations.
- Limits: limits at a point, right and left sided limits, limits at infinity, vertical/horizontal asymptotes as limits and understanding graphical behavior, infinite limits.
- Limit properties and computational techniques.
- Establishing limit properties using the definition of the limit, computing limits using the definition of a limit and limit properties, proving the value of a limit using the definition of the limit.
- Limits of rational functions, indeterminate forms.
- Continuity: intuitive idea of continuity from a graph, graphical forms of points of continuity and discontinuity, translating continuity into limits.
- Intermediate Value Theorem.
- Limits and continuity of trigonometric functions, Squeeze Theorem (prove that $\lim_{x\to 0}\frac{\sin x}{x}=1$ using Squeeze Theorem).

Chapter 3: The Derivative (10 days)

- Tangent lines and Rates of Change: slopes of secant lines, using limit to turn a secant line into a tangent line, distance vs. time graph and secant line (slope is average velocity) and tangent line (slope is "instantaneous" velocity), average rates of change.
- Definition of the derivative: algebraic representation using limit of the difference quotient and connection to tangent line on a graph. Discuss interpretation as instantaneous rate of change.
- Differentiability.
- Connection between differentiability and continuity.
- Methods of Differentiating: constants, power rule, sum and difference of functions, product and quotient of functions (prove these using the limit definition of derivative and properties of limits).
- Derivative at a point and as a function using limit definition of derivative and methods of differentiating.
- High Derivatives: 2^{nd}, 3^{rd}, and n^{th} derivative. Many forms of derivative notation.
- Derivative of Trig functions (prove using limit definition of derivative and special trig limits/properties)
- Chain Rule and u-substitution, differentiating with respect to particular variables.
- Local Linear Approximation: application problems using data values and known rates of change.
- Approximating rates of change from graphs and tables.

Chapter 4: Inverse, Logarithmic, and Exponential Functions (12 days)

- Inverse Functions: definition of inverse functions, examine domain and range, finding existence of, graphing by hand and on graphing calculator, existence of inverse when function is increasing or decreasing, restricting domains to make function invertible, vertical & horizontal line tests.
- Continuity and differentiability of inverse functions: how to determine graphically and algebraically (corners, discontinuity, horizontal/vertical tangents, etc) and connection between representations.
- Logarithmic and Exponential Functions: connection as inverses, graphing, domain and range, algebraic properties, solving logarithmic equations, creation of change of base formula, limits of logarithmic and exponential functions.
- Implicit Differentiation: connections to inverse, finding slope of tangent for functions too complex to rewrite explicitly.
- Activity "Calculator Exploration: Discovering the Natural Number *e*" (see attachment).
- Definition of natural number *e* using limits.
- Derivatives of logarithmic and exponential functions (prove $\log_b x$ using limit definition of derivative. Prove derivative of $\ln(x)$ using $\log_e x$ and previous result. Prove derivative of $y = b^x$ using logarithmic differentiation).
- Defining Inverse Trig Functions: restricting domain looking at graph, evaluating inverse trig expressions without a calculator using unit circle in coordinate plane ("the butterfly").
- Derivative of Inverse Trig Functions.

- Related Rates: recall derivative as a rate of change (slope of tangent), consider position (s) vs. time (t) slope of tangent $\frac{ds}{dt}$.
- L'Hôpital's Rule and Indeterminate Forms: consider as a fraction's numerator and denominator both go to zero during a limit (we know from before that limit can still exist), similarly with infinity over infinity. Algebraically changing $0 \cdot \infty$ into either $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Chapter 5: Using the Tools of Calculus (5 days)

- Analysis of Function: examine the graphs of functions and determine increasing and decreasing in relationship to derivative (slope of tangent line), examine graphs to determine concavity in relationship to first derivative and second derivative. Find intervals of increasing and decreasing analytically through derivative (no graph), and intervals of concave up and down analytically through first and/or second derivative (no graph) and points of inflection. Mathematical solutions must be justified verbally, explaining how the math indicates the conclusion.
- Further Analysis of Functions: relative extrema (max and min), examine graphs and determine where local (or even absolute) max and min occur (different critical values: stationary, non-differentiable, or endpoint for absolute extrema), First Derivative Test, Second Derivative Test. Mathematical solutions must be justified verbally, explaining how the math indicates the conclusion.
- Using analysis of a function to sketch a graph of function (tools of calculus): zeros, horizontal asymptotes (end behavior) and vertical asymptotes, critical values/points, relative extrema, intervals of increase/decrease, inflection points, and intervals of concave up/down.

Chapter 6: Application of the Derivative (10 days)

- Absolute Maxima and Minima: consider largest and smallest relative extrema and endpoints (if finite interval)
- Extreme Value Theorem and the existence of absolute max/min on open/infinite interval.
- Applied Max and Min: various geometric, physics, economic, and other application problems.
- Rectilinear Motion: recall derivative is instantaneous rate of change, situations of (+) or (–) values of position, velocity, and acceleration, consider language use of "speeding up" and "slowing down" and frame of reference. Looking at a graph of position vs. time OR velocity vs. time OR acceleration vs. time OR speed vs. time and determine things about position, velocity, acceleration, speeding up/down, direction of travel, etc. Determine these things also looking at a table of values. Mathematical solutions must be justified verbally, explaining how the math indicates the conclusion.
- Solve application word problems involving position, velocity/speed, and acceleration using tools of calculus.
- Rolle's Theorem: graphically see what Rolle's Theorem says and understand which conditions are necessary.

- Mean-Value Theorem (for derivatives): make connection to Rolle's Theorem (Rolle's specific, Mean-Value generic), graphically/geometrically see what Mean-Value Theorem says and understand which conditions are necessary. Applying Mean-Value Theorem to data in a table, graph of a function, or algebraic equation to determine further information.

Chapter 7: The Integral
Indefinite Integral and Definite Integral (10 days)

- Indefinite Integral: look at indefinite integral as anti-derivative, consider family of functions that could be the anti-derivative (constant +C), notation of indefinite integral, properties of indefinite integral.
- Techniques of integration (part 1): integration by substitution, undoing simple chain rule (look for the derivative of what's inside "eyeball method").
- Induction (a tool used to introduce Riemann Sums, the definite integral, and area under curve using rectangles): use induction to prove formulas like $\sum_{k=1}^{n} k = 1+2+3+...+n = \frac{n(n+1)}{2}$ and $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+...+n^3 = \left[\frac{n(n+1)}{2}\right]^2$.
- Area Under the Curve using Rectangle: graphically see that area under curve can be approximated using rectangles (left-hand, right-hand, and midpoint rectangles, and also trapezoids), conceptualize that if we have an infinite number of infinitely thin rectangles then the error is eliminated and actual area is achieved, write a formula for the sum of *n* rectangles (use sum formulas proved by induction!) and then take limit as *n* goes to infinity and get actual area.
- Riemann Sums: Riemann Sums concept, compare to sum of area of rectangles approximating area under curve previously done, define the definite integral as the limit of Riemann Sum, introduce notation (do not know how to evaluate notation yet, only interpret as area under curve!).
- Properties of Definite Integral: consider properties of definite integral by looking at area under curve(s) on graphs $\int_a^a f(x)dx = 0$, $\int_b^a f(x)dx = -\int_a^b f(x)dx$, $\int_a^b c \cdot f(x)dx$, $\int_a^b f(x) \pm g(x)dx$, $\int_a^c f(x)dx + \int_c^b f(x)dx$.
- Fundamental Theorem of Calculus (FTC): prove both parts of the FTC, make connection between definite integral and the indefinite integral (anti-derivative), method of evaluating definite integrals.
- Mean-Value Theorem (for Integrals): graphically see Mean-Value Theorem as average area that lies between area created from one large rectangle and the area created from one small rectangle. Finding average value of a function using the Mean-Value Theorem for integrals.
- Rectilinear Motion from the point of view of the integral. Integrating rates of change equals total change, finding total distance traveled and finding displacement (compare the two).

Chapter 8: Application of the Integral (13 days)

- Area between two curves (compare to Area “under” the curve).
- Integration as net accumulation of change.
- Application of area between curves and rectilinear motion: what does area under curve represent? What does area between curve represent? Students explain verbally.
- Volume of a rotation: emphasis on creating Riemann Sums and being able to set up any number of situations, integrating with respect to horizontal or vertical, using “disk” method, “washers” method, and “cylindrical shells” method.
- Volume of solid by projecting perpendicular cross-sections.
- Activity: Volume of Solid Project. (see attachment)
- Length of a plane curve: emphasize Riemann Sum of infinitely many points.
- Surface Area of revolution using length of curve Riemann Sums.
- Applied Physics problems: Work, and Hooke’s Law, pumping water out, work stretching a spring, work done pulling in a chain. Emphasize creating Riemann Sums.
- Applied Physics problems (continued): Fluid Pressure and Fluid Force. Finding total fluid force on submerged objects.
- Demonstration of Fluid Force/Pressure.

Chapter 9: Techniques of Integration (10 days)

- Integration By Parts: creating integration by parts after considering undoing product rule. Sometimes doing by parts multiple times and doing algebra with integral expressions. (Time permitting)
- Trigonometric Integrals: using trig identities to simplify in order to integrate.
- Trigonometric Substitution: how to undo the derivative of a trig inverse.
- Partial Fractions. (Time permitting)
- Improper Integrals: integrating with infinite discontinuities and over infinite intervals. First consider graphical representation, then look at table representation of data. Can have a definite/finite answer of integral! Illustrate methods to tell if definite integral converges or diverges. (Time permitting)
- Simpson’s Rule (done using C++ programming).

Chapter 10: Mathematical Modeling with Differential Equations (4 days)

- First-Order Separable Differential Equations: solving differential equations and finding a general/family of solutions (look at graphical examples of family of solutions), finding particular solutions using an initial condition (i.e. the initial value problem) (look at graphical examples of a particular solution selected from a family of solutions), solving initial value problem for NON-separable differential equations.
- Modeling with Differential Equations: Exponential Growth and Decay models created from verbal situations (“when the rate of growth/decay is proportional to the amount of the quantity

present" – the initial value problem that leads to exponential growth/decay model), Logistic Growth Model (inhibited population model).

- Slope Field Activity. (see attachment)
- Slope Fields: discuss slope field as a direction field giving us a picture of how different solutions look (they must flow through the slope field), use slope field to approximate solutions to differential equations that are too difficult to solve (non-separable, non-linear, just too difficult), match slope fields to possible solutions (students must justify verbally why they have made the match).
- Euler's Method: using only differential equation and an initial condition (the initial value problem) we can approximate values of a solution. Use Euler's Method if we are unable to find the actual solution to the differential equation. Compare and contrast with local linear approximation. Illustrate use in real world problems (real world has you look at data and rates of change, which are measurable quantities, and then determine values from that).

Teaching Strategies:

It is extremely important to understand the motivation and development of calculus concepts. In calculus, this often includes the use of graphs. Some examples of concepts that are first introduced by analyzing the graphical representation include:

- Derivative (slope of tangent line)
- Definite integral (area under the curve, volume of revolution)
- Trigonometric functions (the unit circle in coordinate plane, quadrants and the "butterfly")
- Limits (physically getting closer and closer)
- Continuity (intuitive graphical meaning)
- Local linear approximation (tangent line is "locally" fairly close to the graph of a function)
- Relative and absolute extrema (seeing tangent line go from positive, to zero, and to negative or visa-versa)
- The Mean-Value Theorem and Rolle's Theorem (considering slopes of secant lines and tangent lines)
- The Mean-Value Theorem for Integrals (considering the area of rectangles and the actual area under the curve)
- Differential equations (slope fields and the distinction between family of solutions and a particular solution)
- and many others…